NYUAD **Calculus With Applications** **Fall Term 2013**

Arman Mimar Office Sama 1309 am623@nyu.edu
Erwan Biland Office Sama 1310 erwan.biland@nyu.edu

Final Exam - Dec 16, 2013

First name:
Last Name:
ID:

Remarks: A problem will be deemed correctly solved, only when it is fully justified. Without such justification, even a correct answer will be granted limited credit. Thus, it is important to show your work in all the questions, and be clear about why you reach your conclusions.
No calculators of any kind during the exam.
You may answer all 10 questions on the exam. However, any 9 correctly answered questions will get you more than 100% of the grade.

Problem	Possible	Points
1	12	
2	12	
3	12	
4	12	
5	12	
6	12	
7	12	
8	12	
9	12	
10	12	
Total	120	

1. Calculate the following limits (if they do not exist, explain why):

$$\lim_{n\to\infty} \frac{n^5+1}{n!-1} \qquad ; \qquad \lim_{x\to 0} \frac{\ln(1+x^2)}{x^2} \ .$$

[For the second limit, you may use L'Hospital's rule.]

2. Let the function f be defined by $f(x) = \ln(\arcsin\sqrt{3x+1})$.

a) Find the domain of f.

b) Compute the derivative of f. *[You may use that* $\arcsin'(t) = \dfrac{1}{\sqrt{1-t^2}}$.*]*

3. Find the absolute minimum and maximum values of the function:

$$f : [0, 10] \to \mathbb{R}, \quad t \mapsto x^3 - 9x^2 + 15x.$$

4. Use integration by parts to compute the indefinite integrals:

$$\int x\, e^{-3x}\, dx \qquad ; \qquad \int x \arctan x \, dx \ .$$

[For the second integration by parts, you should use $\frac{1}{2}(1+x^2)$ *as an antiderivative of* x*.]*

5. Is the following improper integral convergent? If so, compute its value.

$$\int_1^\infty \frac{\ln x}{x^2}\, dx$$

6. Use the substitution rule to compute the following integrals:

$$\int_0^1 x\, e^{-x^2}\, dx \qquad ; \qquad \int_0^{\pi/2} \frac{\cos\theta}{2+\sin\theta}\, d\theta \ .$$

7. Are the following series convergent or divergent?

$$\sum_{n \geqslant 2} \frac{5}{n^4} + \frac{4}{n\sqrt{n}} \qquad ; \qquad \sum_{n \geqslant 0} \frac{n^2}{5n^2 - n + 3}$$

8. a) Find the radius of convergence of the power series $\sum_{n\geqslant 0} \frac{(-5)^n}{\sqrt{n+1}}\, x^n$

b) Specify precisely the interval of convergence of this power series (find the exact behaviour at the boundary points).

9. Let the function f be defined by $f(x) = \ln(1 + x^2)$.

a) Calculate $f'(x)$. Express $\dfrac{1}{1+x^2}$ as a power series, and then express $f'(x)$ as a power series.

b) By integration, deduce the power series expansion of $f(x)$ at $a = 0$. Specify the constant.

10. Let the function f be defined by $f(x) = \int_x^{x^3} e^{-t^2}\,dt$. Do not try to compute $f(x)$.

a) Is the function f even, odd or neither? *[Use the substitution $u = -t$.]*

b) Compute the derivative $f'(x)$. Give the values of $f(0)$, $f(1)$, and $f'(0)$.

c) What is the sign of $f(x)$ for $x \in (0, 1)$? for $x > 1$?

d) For $x > 1$, prove that $0 \leqslant f(x) \leqslant (x^3 - x)e^{-x^2}$. Deduce $\lim_{x\to\infty} f(x)$. *[If you did not answer the first part of the question, you can still use it to answer the second part.]*

e) Use the results of questions a-d to sketch the graph of f on $(-\infty, \infty)$.