NYUAD Calculus With Applications Fall Term 2013

Final Exam - Dec 16, 2013

First name:
Last Name:
ID:

Remarks: A problem will be deemed correctly solved, only when it is fully justified. Without such justification, even a correct answer will be granted limited credit. Thus, it is important to show your work in all the questions, and be clear about why you reach your conclusions.
No calculators of any kind during the exam.

Problem	Possible	Points
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
Total	100	

1. Calculate the following limits (if they do not exist, explain why):

$$\lim_{n\to\infty} \frac{n\cos(n)}{\sqrt{(2n)!}} \qquad ; \qquad \lim_{x\to 0} \frac{\sin(x)^2 - x^2}{x^3}$$

2. Let the function f be defined by $f(x) = \dfrac{\sin(x)\cos^2(x)}{x^5 \ln(x)}$.

a) Find the domain of f.

b) Compute the derivative of f.

3. Find all critical points, local and global minima and maxima for the function:

$$f : [0, 10] \to \mathbb{R}, \quad x \mapsto \frac{x}{1 + x^2} + \frac{x}{20}$$

4. Use integration by parts to compute the indefinite integrals:

$$\int \ln(x)\, x^3\, dx \qquad ; \qquad \int x \cos(2x)\, dx$$

5. Is the following improper integral convergent?

$$\int_0^\infty x^2\, e^{-x^2}\, dx$$

6. Use the substitution rule to compute the following integrals:

$$\int_0^1 \frac{1}{\sqrt{x+2}}\,dx \qquad ; \qquad \int_0^5 \frac{x^3+x}{3+x^2}\,dx$$

7. Are the following series convergent or divergent?

$$\sum_{n\geqslant 2} \frac{\sqrt{n}\,\cos(n)}{1+n^2} \qquad ; \qquad \sum_{n\geqslant 1} \frac{n+\ln(n)}{n\sqrt{n+1}}$$

8. Is the following series absolutely convergent, conditionally convergent, or divergent?

$$\sum_{n \geqslant 1} \frac{\sin^2(n) - \cos^2(n)}{n(n+1)}.$$

9. a) Find the radius of convergence of the power series $\sum_{n\geqslant 1} \frac{n+3}{n^3+1}(x+2)^n$

b) Specify precisely the interval of convergence of this power series (find the exact behaviour at the boundary points).

10. Let the function f be defined by $f(x) = \frac{1}{(2+x)^2}$.

a) Express $\dfrac{1}{1+\frac{x}{2}}$ as a power series.

b) By differentiation, deduce a power series expansion of $f(x)$.

c) Use this expansion to calculate $f(0)$, $f'(0)$, $f^{(2)}(0)$ and $f^{(3)}(0)$.