Numerical Methods - Assignment 13

Due Tuesday, Nov 26

1 - Consider the differential equation $y' = y$, on the interval $[0, 1]$, with initial condition $y(0) = 1$. Let $N > 0$ be an integer; write $h = 1/N$ and $x_k = k/N$, $k = 0, \ldots, N$. Let $y_k = y(x_k)$ be the exact solution and $u_k = u(x_k)$ be its approximation obtained by the backward Euler method. Compute explicitely the numbers $u_0, \ldots, u_N$, and give an estimate of the error $\varepsilon(x_k)$ as a function of the step length h.

2 - Do the same with the Crank-Nicolson method.

3 - Do the same with the improved Euler method defined by

$$u_{k+1} = u_k + \frac{h}{2}\left[f(x_k, u_k) + f(x_{k+1}, u^*_{k+1})\right], \qquad \text{where } u^*_{k+1} = u_k + f(x_k, u_k).$$

[This is also known as the *Heun method*.]

Please show your work in details for question 1; you might go faster for the next two questions.