# Numerical Methods - Assignment 15

Due Tuesday, Dec 3

The orbit of a planet around the sun can be approximated by the differential equation

$$\frac{d^2M}{dt^2} = -k\frac{\overrightarrow{OM}}{OM^3},$$

where $M$ is the position of the planet in the ecliptic plane $\mathbb{R}^2$, the sun is fixed at the origin $O$, and $k$ is a constant that depends on the masses of the planet and the sun.

**1 -** Let $x(t), y(t)$ be the coordinates of $M$ at time $t$. Write $u(t) = x'(t), v(t) = y'(t)$. Show that the vector $(x, y, u, v)$ satisfies a first-order differential equation, which can be written as a system of 4 differential equations in terms of the 4 functions $x, y, u, v$.

**2 -** We wish to solve the above equation for $k = 1$, with initial condition $x(0) = 10$, $y(0) = 0$, $x'(0) = 0$, $y'(0) = 10$. Implement a function `Orbit( h , T )` that approximates the solution by the improved Euler method, for a given step $h$, on the interval $[0, T]$ (or rather $[0, Nh]$, where the integer $N$ is such that $Nh \approx T$), and plots the resulting trajectory in the $(x, y)$ plane. The actual solution of the equation is periodic, so the trajectory obtained should be roughly periodic. Choose $T$ such that several periods are plotted, to check that the trajectory seems to be stable.

**3 -** (facultative) You may then modify the previous answer to implement a function `AdaptiveOrbit( eps, hmin , T )` that does the same thing using an adaptive step $h$, with a given relative tolerance $\epsilon h$ at each step, and a given minimum step $h_{min}$.