

**NYUAD**

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**Numerical Methods**

Office Sama 1310

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**Midterm Exam - Practice**

**First name:**

**Last Name:**

**ID:**

**Remarks: You may use your computer anytime. The use of Matlab is required for some questions.**

<b>Problem</b>	<b>Possible</b>	<b>Points</b>
<b>1</b>	<b>20</b>	
<b>2</b>	<b>20</b>	
<b>3</b>	<b>20</b>	
<b>4</b>	<b>20</b>	
<b>5</b>	<b>20</b>	
<b>Total</b>	<b>100</b>	

1. a) Define the subordinate norm of an  $n \times n$  matrix  $A$  with respect to the  $\infty$ -norm  $\|\cdot\|_\infty$  on  $\mathbb{R}^n$ .

b) Compute  $\|A\|_\infty$  for  $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$  (no proof is required).

2. Let  $\langle \cdot, \cdot \rangle$  be an inner product on an  $\mathbb{R}$ -vector space  $V$ . Let  $(x_1, \dots, x_n)$  be a family of linearly independent vectors in  $V$ . Explain the Gram-Schmidt method to turn this family into an orthonormal family  $(e_1, \dots, e_n)$ . You must include any formula that is needed.

**3. Compute with Matlab the LU factorisation of the matrix**

$$B = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 0 \\ 6 & 7 & 8 & 9 & 1 & 2 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 2 & 1 & 9 & 8 & 7 & 6 \\ 0 & 5 & 4 & 3 & 2 & 1 \end{bmatrix} .$$

4. a) Give the formula of the composite Simpson rule with  $n$  subintervals to approximate  $\int_a^b f(t)dt$ .

b) Use Matlab to compute  $\int_0^5 \frac{1-x^2}{1+x^2}$  with 10 digits.

5. a) Write down a Matlab program that computes an approximation of the solution to the equation  $x^5 + x^2 = 1000$  with error smaller than  $10^{-9}$ .

b) Write down the result returned by your program.